BROWN-HET-995
April 1995

ACCRETION OF HOT DARK MATTER ONTO SLOWLY MOVING COSMIC STRINGS

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ABSTRACT

Cosmic strings with small-scale structure have a coarse-grained mass per unit length μ which is larger than the string tension. This leads to an effective Newtonian gravitational line source and to a characteristic translational velocity which is smaller than for strings without small-scale structure. Here, the accretion of hot dark matter onto such strings is studied by means of the Zel'dovich approximation. We find that clustering is greatly enhanced by the Newtonian line source. In the limit of vanishing translational velocity, the first nonlinear filaments form at a redshift of greater than 100 for standard values of μ.

1. Introduction

A theory in which the dark matter is hot and in which the primordial density perturbations are seeded by cosmic strings is a viable alternative[1,2] to the currently popular variants of cold dark matter (CDM) cosmologies with adiabatic density fluctuations produced during a hypothetical period of inflation in the very early Universe. On large length scales, the theory predicts a scale-invariant spectrum of perturbations. The normalizations of the model from large-scale structure[3] and from cosmic microwave background anisotropy measurements[4] are in encouraging agreement. The theory also predicts that on scales comparable to and larger than the comoving horizon at t_{eq}, the time of equal matter and radiation, galaxies are concentrated in sheets or filaments[3,5–8], in agreement with the results from the Center for Astrophysics galaxy redshift survey[9].

If the primordial density inhomogeneities are nonadiabatic, slowly decaying seeds such as cosmic strings, hot dark matter (HDM) is a viable dark matter candidate. In contrast to adiabatic inflationary models in which free streaming erases perturbations on all scales smaller than the maximal free streaming length λ_J^{max}, the strings survive neutrino free streaming[10] and can at late times seed structures on scales much smaller than λ_J^{max}. There is[11] still power on scales smaller than λ_J^{max} (although less than in the inflation-based CDM cosmology), and the power spectrum may well be in better agreement with the results from recent observations[12] (the decrease in power on small scales compared to the inflationary CDM model is a promising feature).

In spite of the abovementioned advantages of the cosmic string theory, there is still a lot of ignorance about the specific predictions of the model. One important reason is that the initial conditions for structure formation in the theory are not yet accurately known. It is well established that the network of cosmic strings (and thus the distribution of the seeds for density inhomogeneities) approaches a scale-invariant distribution, but the detailed properties of this distribution are uncertain. It can be shown that at all times t, there are a few strings which are

straight on a correlation length $\xi(t) \sim t$ crossing each Hubble volume, but the small-scale structure of these strings is not known. Also uncertain is the exact number of such strings per Hubble volume.

The evolution of the cosmic string network may leave behind on the strings a lot of structure (kinks and wiggles) on scales smaller than ξ. The typical amplitude of these perturbations is much smaller than ξ, and hence in a coarse grained description, the strings can still be viewed as straight on a scale ξ, but with renormalized mass per unit length and tension. Since the integrated length of the string between two points is longer than the separation between the points, the coarse grained mass per unit length is larger than the microscopic length. At the same time, the coarse grained tension is less than the microscopic tension. As first discussed by Carter[13)] and subsequently analyzed in detail by Vollick[6)] and by Vachaspati and Vilenkin[7)], small-scale structure on strings can lead to substantial modifications of the motion and gravitational effects of the strings.

A long straight string without small-scale structure induces no local gravitational force. However, space perpendicular to the string is a cone with deficit angle[14)]

$$\alpha = 8\pi G\mu, \tag{1.1}$$

where μ is the mass per unit length of the string coarse grained on a scale ξ and G is Newton's constant. When moving with a transverse speed v_s, such a string will induce a velocity perturbation of magnitude

$$\delta v = 4\pi G\mu v_s \gamma_s \tag{1.2}$$

towards the plane behind the string, leading to the formation of a "wake", a planar density enhancement[5)]. In the above, γ_s is the relativistic γ factor associated with v_s.

A long straight string with small-scale structure will induce, in addition to the deficit angle given by Eq. (1.1), a nonvanishing local gravitational force of

magnitude[6,7)]

$$F = 2mG(\mu - T)/r \tag{1.3}$$

towards the string when acting on a test particle of mass m located a distance r from the string. The string tension is T, and T is related to μ by[13)]

$$\mu T = \mu_0^2, \tag{1.4}$$

where μ_0 is the microscopic mass per unit length of the string. In addition, when coarse grained on a scale of ξ, strings with $\mu > \mu_0$ have a smaller transverse velocity than those with $\mu = \mu_0$. In the limit when the distance the string moves in one Hubble expansion time is small compared to the thickness of the wake, the structure seeded by the string becomes filamentary.

It is important to know at what redshift the first density perturbations become nonlinear (this gives an upper bound to the redshift of first star formation). To answer this question in our theory, the clustering of HDM induced by cosmic strings must be studied.

The accretion of HDM onto cosmic string loops was studied in detail in Ref. 10. It was shown that free streaming slows down but never reverses the growth of perturbations on small scales. Structures form "inside-out" in the sense that the first shells to go nonlinear around the seed are the innermost ones. In a toy model in which only cosmic string loops are present as seeds, this leads to a "bottom-up" scenario for structure formation. The first nonlinear objects form at high redshifts.

However, the current cosmic string simulations[15)] indicate that most of the mass in the string network resides in the long strings (strings with curvature radius $\xi(t) \sim t$ at cosmic time t). Hence, in order to determine the earliest time at which large-scale nonlinear structures can form, it is necessary to study the clustering of HDM induced by long strings. This was done in Ref. 3 for strings without small-scale structure. The results were very different than for loops. Planar collapse of HDM does not proceed in an "inside-out" fashion. Rather, the first sheet to go

nonlinear has an initial comoving distance $q_{max} \sim 1$ Mpc from the center of the wake, and the onset of nonlinearity occurs late, i.e. at a redshift $z_{max} \sim 1$ for values $G\mu \sim 10^{-6}$ (these results will be reviewed at the end of Section 2).

In this paper, we study the accretion of HDM onto long strings with nonvanishing small-scale structure (which leads to the presence of a Newtonian gravitational line source). The calculations will be done in the limit in which the tranlational motion of the string can be neglected.

We find that in this situation, structures once again form "inside-out", and that the first nonlinear structures emerge at a high redshift ($z_{max} > 10^2$ for $G\mu \sim 10^{-6}$). This result indicates that in a cosmic string theory with HDM, there will be no problem in explaining the recent observations of high redshift clusters[16)], quasars[17)] and QSO absorption line systems[18)].

In the following section we summarize our calculations, and in Section 3 we discuss some implications of our results. Units in which $c = k_B = 1$ are used throughout. The scale factor of the Universe is denoted by $a(t)$, t being cosmic time, and the associated redshift is $z(t)$. The present time is t_0.

2. Accretion of Hot Dark Matter onto a Line Source

We will study the accretion of collisionless hot dark matter (HDM) onto a stationary cosmic string using an adaptation of the Zel'dovich approximation[19)] to HDM. This modification was applied to planar collapse in Ref. 3 and was shown to give good agreement with the results of a linearized Boltzmann equation method (which takes into account the full phase space distribution of HDM particles; see Refs. 3 & 10).

HDM particles have near-relativistic velocities $v(t)$ at times close to t_{eq}. This prevents them from clustering on length scales smaller than their free streaming distance

$$\lambda_J(t) \simeq 3v(t)z(t)t, \tag{2.1}$$

the average comoving distance a particle will move in a Hubble expansion time. Note that since $v(t) \propto z(t) \propto a(t)^{-1}$, the free streaming length decreases in time as

$$\lambda_J(t) \propto t^{-1/3} \tag{2.2}$$

and takes on its maximal value λ_J^{max} at t_{eq}. In a spatially flat HDM Universe

$$v(t_{eq}) = v_{eq} \simeq 0.1 \tag{2.3}$$

and hence

$$\lambda_J^{max} \simeq 3h^{-2}Mpc, \tag{2.4}$$

substantially larger than the typical size of a galaxy.

We will first describe the Zel'dovich approximation for filamentary accretion and then in a second step adapt the method to HDM. Without loss of generality we can choose coordinates in which the string lies along the z coordinate axis. We consider the response of the dark matter particles to the gravitational line source produced by the small-scale structure of the string, focusing on the "plane" (strictly speaking a cone) at $z = 0$. Let $\vec{q}$ denote the initial comoving separation of a dark matter particle from the string. In the absence of the string line source, the physical separation $\vec{r}(t)$ would grow as the Universe expands. The presence of the string, however, leads to a comoving displacement $\vec{\psi}(\vec{q}, t)$ which grows in time:

$$\vec{r}(\vec{q}, t) = a(t)(\vec{q} - \vec{\psi}(\vec{q}, t)). \tag{2.5}$$

The Zel'dovich approximation consists of taking the Newtonian equations

$$\ddot{\vec{r}} = -\frac{\partial}{\partial \vec{r}} \Phi(\vec{r}, t) \tag{2.6}$$

for $\vec{r}(\vec{q}, t)$ with the gravitational potential $\Phi(\vec{r}, t)$ determined by the Poisson equa-

tion

$$\frac{\partial^2}{\partial \vec{r}^2}\Phi = 4\pi G\left(\rho(\vec{r},t) + \frac{\lambda\delta(r)}{2\pi r}\right) \tag{2.7}$$

and linearizing in ψ. On the right hand side of the Poisson equation, the total density has been separated into the dark matter density $\rho(\vec{r},t)$ and the contribution of the string-induced line source with an effective mass per unit length

$$\lambda = \mu - T \tag{2.8}$$

(see Eq. (1.3)), and where $\delta(r)$ denotes a Dirac delta function in cylindrical coordinates. The density $\rho(\vec{r},t)$ is in turn determined in terms of the background density $\rho_0(t)$ by the mass conservation equation

$$\rho(\vec{r},t)d^3\vec{r} = a^3(t)\rho_0 d^3\vec{q} \tag{2.9}$$

which gives

$$\rho(\vec{r},t) = a^3(t)\rho_0 det^{-1}\left|\frac{d\vec{r}}{d\vec{q}}\right| = a(t)\rho_0(\frac{dr}{dq})^{-1} \simeq \rho_0(1 + \frac{\partial}{\partial q}\psi_r(q,t)), \tag{2.10}$$

where r, q and ψ_r are (in cylindrical coordinates) the radial components of $\vec{r}$, $\vec{q}$ and $\vec{\psi}$, respectively, and where in the final step we have linearized in $\vec{\psi}$.

Inserting (2.10) into (2.7) and integrating once gives

$$\frac{\partial}{\partial \vec{r}}\Phi = 4\pi G\left[\frac{1}{3}\rho_0(t)\vec{r} + \rho_0(t)a(t)\vec{\psi}(\vec{q},t) + \frac{\lambda\theta(r)\hat{r}}{2\pi r}\right], \tag{2.11}$$

which we insert on the right hand side of (2.6). Note that $\hat{r}$ is the unit radial vector in the plane perdendicular to the string. On the left hand side of (2.6) we

substitute the second time derivative of (2.5),

$$-\ddot{\vec{r}} = a\ddot{\vec{\psi}} + 2\dot{a}\dot{\vec{\psi}} - \ddot{a}\vec{q} + \ddot{a}\vec{\psi}. \tag{2.12}$$

Using the FRW equation $\ddot{a} = -\frac{4\pi G}{3}\rho_0 a$ (valid in the matter dominated era) to substitute for $\rho_0 a$ in (2.11), we obtain as our basic equation

$$\ddot{\psi}_r + 2\frac{\dot{a}}{a}\dot{\psi}_r + 3\frac{\ddot{a}}{a}\psi_r = \frac{2\lambda G}{a^2(q-\psi_r)}. \tag{2.13}$$

This equation is only valid for $t > t_{eq}$, since we made use of an approximate pressureless equation of state. Hence, we should also substitute $a(t) = (t/t_0)^{2/3}$ and then obtain

$$\ddot{\psi}_r + \frac{4}{3}t^{-1}\dot{\psi}_r - \frac{2}{3}t^{-2}\psi_r = \frac{2\lambda G t_0^{4/3}}{t^{4/3}(q-\psi_r)} \tag{2.14}$$

as equation of motion for the radial displacement ψ_r.

A second reason why the above derivation is only valid for $t > t_{eq}$ is that the presence of the uniform radiation bath has been neglected. This radiation would have to be considered in the Poisson equation (2.7) and would stall the growth of perturbations before t_{eq}. Hence, for a cosmic string theory with cold dark matter, the initial conditions for solving (2.14) are

$$\dot{\vec{\psi}}(t_s) = \vec{\psi}(t_s) = 0, \tag{2.15}$$

with $t_s = t_{eq}$ (for strings present at t_{eq}) and $t_s = t_i$ for strings "appearing" at a later time.

For a static string network, it does not make sense to consider $t_i > t_{eq}$. However, for a dynamical string network, $t_i > t_{eq}$ would apply to string segments whose curvature radius ξ is equal to the Hubble radius at time t_i and thus start their proper motion at that time.

To first order in perturbation theory, we solve (2.14) using the Born approximation

$$\ddot{\psi}_r + \frac{4}{3}t^{-1}\dot{\psi}_r - \frac{2}{3}t^{-2}\psi_r = \frac{2\lambda t_o^{4/3}G}{qt^{4/3}}. \tag{2.16}$$

This can be solved using the Green's function method which gives

$$\psi_r(t) = u_1(t)\int\limits_{t_s}^{t} dt'\epsilon(t')u_2(t')f(t') - u_2(t)\int\limits_{t_s}^{t} dt'\epsilon(t')u_1(t')f(t'), \tag{2.17}$$

where $u_1(t) = (\frac{t}{t_0})^{2/3}$ and $u_2(t) = (\frac{t}{t_0})^{-1}$ are the two fundamental solutions of the homogeneous equation, $\epsilon(t)$ is the Wronskian

$$\epsilon(t) = (\dot{u}_1 u_2 - \dot{u}_2 u_1)^{-1}, \tag{2.18}$$

and

$$f(t) = \frac{2\lambda G}{q}\left(\frac{t_0}{t}\right)^{4/3}. \tag{2.19}$$

Some manipulations give the end result:

$$\psi_r(t) = \frac{6\lambda G}{5q}t_0^{4/3}t^{2/3}\left[ln(t/ts) - \frac{3}{5}\left(1 - \left(\frac{t_s}{t}\right)^{5/3}\right)\right]. \tag{2.20}$$

The real objective of the calculation, however, is to find the comoving scale q_{nl} which has gone non-linear at a given time t, as a function of the time of formation t_i. This scale is defined as the scale at which at a time t the physical velocity of a particle $\dot{r}(q_{nl}, t)$ vanishes, giving an equation for q_{nl} of

$$\dot{a}(t)(q_{nl}(t_s, t) - \psi(t)) = a(t)\dot{\psi}(t). \tag{2.21}$$

Together with Eq. (2.20) and the assumption that $a(t) = (t/t_0)^{2/3}$, this gives (after

a fair amount of algebra)

$$q_{nl}(t_s,t) = t^{1/3} t_0^{2/3} \sqrt{\frac{12G\lambda}{5}\left[log(t/t_s) - \frac{3}{20}\left(\left(\frac{t_s}{t}\right)^{5/3} - 1\right)\right]}. \tag{2.22}$$

For CDM, this equation gives the size of the non-linear region at time t for a structure which starts to grow at time t_s.

Because of neutrino free streaming, the above formalism is not directly applicable to the clustering of HDM. A rigorous analysis must follow the entire phase space distribution of the dark matter. Starting point is the collisionless Boltzmann equation describing the time evolution of the phase space density, expanded perturbatively about a homogeneous thermal distribution. After integrating over momenta one obtains the Gilbert equation[20)], an integral equation for the configuration space density perturbation. However, in Ref. 3 it was shown that the Zel'dovich approximation can easily be adapted to HDM and then gives good agreement with the results from the Gilbert equation approach.

The modified Zel'dovich approximation consists of using the same dynamical equation (2.14), but with initial conditions

$$\dot{\vec{\psi}}(t_s(q)) = \vec{\psi}(t_s(q)) = 0, \tag{2.23}$$

where $t_s(q)$ is the time when the neutrino free streaming length $\lambda_J(t)$ equals q, i.e.

$$\lambda_J(t_s(q)) = q. \tag{2.24}$$

Solving for $t_s(q)$ gives

$$t_s = \frac{t_0 \lambda_0^3}{q^3}, \qquad \lambda_0 = 3v_{eq} z_{eq}^{1/2} t_{eq}. \tag{2.25}$$

Note that if (2.24) gives a value of t smaller than t_i, then $t_s(q)$ must again be chosen to be t_i.

For HDM, turnaround in determined by Equation (2.22), but with the time t_s depending on q as given by (2.25). Inserting (2.25) into (2.22) and solving for q at late times gives

$$\frac{6G\lambda t_0^2}{5z(t)}\left(6log\left(\frac{q}{z^{1/2}(t)\lambda_0}\right)+\frac{3}{10}\right)-q^2=0. \tag{2.26}$$

This implicit equation gives, for a given scale q, the redshift $z(t)$ at which this scale q first goes non-linear. This equation only holds for values of q smaller than q_{max}, the value of q for which $t_s(q)$ equals t_i, i.e.

$$q_{max}=\lambda_0 z^{1/2}(t_i). \tag{2.27}$$

For q larger than q_{max}, Equation (2.25) with $t_s=t_i$ must be used.

Equation (2.26) can be solved numerically. The result is shown in Figure 1. The main conclusion is that in the filament model with HDM, non-linear regions can develop at very early times - at redshifts larger than 10^2 for the usual cosmic string normalization $G\mu=10^{-6}$. The first scales to go non-linear are small scales.

These results contrast with those for planar collapse of HDM[3)], where no non-linearities appear until a redshift of about 1, and the first scale to turn around is $q=q_{max}$ (see Figure 2).

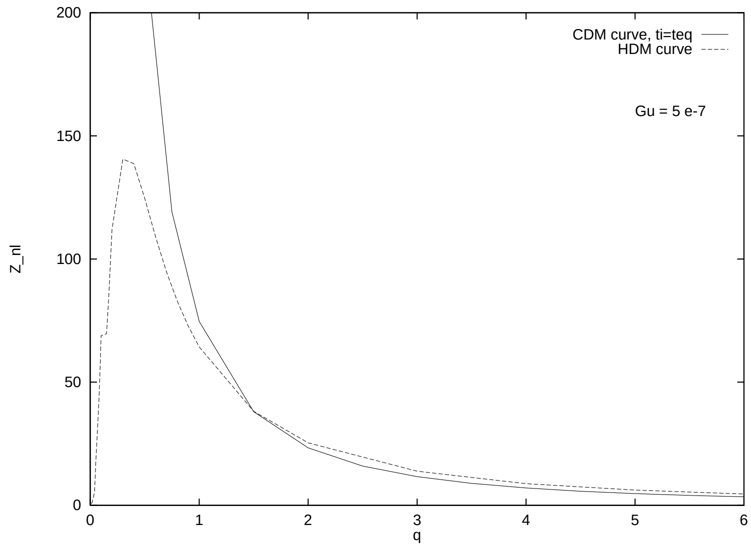

Figure 1: Redshift z_{nl} at which a scale q (in Mpc) turns around for filamentary accretion in the case of HDM (dashed line) and CDM (solid line). The value of $G\lambda$ was $5 \cdot 10^{-7}$, and the case $t_i = t_{eq}$ was considered.

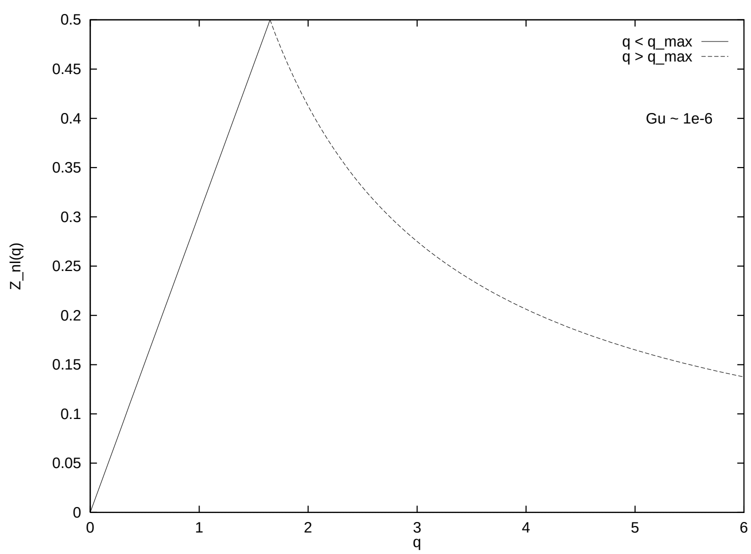

Figure 2: Redshift z_{nl} versus scale q (in Mpc) for accretion of HDM onto a cosmic string wake for a value $G\mu = 10^{-6}$ (from Ref. 3).

3. Discussion

We have studied the accretion of HDM onto a stationary cosmic string with nonnegligible small-scale structure. Such a string acts as a Newtonian line source of gravity for HDM. Our main results are summarized in Figure 1: the first scales to turn around are small scales, and the redshift when the first non-linearities appear exceeds 10^2 for the usual cosmic string normalization. This contrasts with the results of planar collapse of HDM onto cosmic string-induced wakes, in which case no non-linear structures form until a reshift of about 1, and where large scales $q = q_{max}$ are the first to turn around.

These results are important in determining the redshift when the first non-linear filamentary structures form in a cosmic string model with HDM. The mass of a filament seeded at t_{eq} by a horizon-length string is

$$M(z) \sim \rho_0 t_{eq} z(t_{eq}) q_{nl}^2(z) \sim 10^{12} M_O (\frac{q_{nl}(z)}{Mpc})^2, \tag{3.1}$$

where ρ_0 is the background energy density and M_O is a solar mass. Such a mass is large enough to be the host of a quasar at a redshift much greater than 1[21]. Hence, explaining the origin of high redshift quasars and galaxies seems not to be a problem in this model.

Obviously, much more work is needed in order to be able to reliably estimate the high redshift mass function in the cosmic string model with HDM. Besides the filamentary accretion mechanism discussed in this paper, there is also accretion at high redshift onto cosmic string loops[10]. Which of the mechanisms is dominant unfortunately depends very sensitively on the parameters in the cosmic string scaling solutions which are not yet reliably known.

Considering a long cosmic string as stationary in order to discuss the accretion of HDM on a given comoving scale q will only be a reasonable approximation if q is larger than the comoving distance the string moves at t_i in one Hubble expansion

time, i.e.

$$q > t_0 z(t_i)^{-1/2} v_s, \tag{3.2}$$

where v_s is the transverse velocity of the string. For $t_i = t_{eq}$, this gives

$$q > 30 v_s Mpc. \tag{3.3}$$

For strings with $v_s \propto 0.03$, the approximation of filamentary accretion is hence reasonable for scales $q \propto 1Mpc$. Note that the r.m.s. value of v_s is about 0.15 in the current cosmic string simulations[15)] which probably underestimate the effects of small-scale structure and thus overestimate v_s.

In conclusion, we believe our results are good news for the cosmic string model with HDM and should encourage further research. The first non-linear structures form early enough in order to explain the origin of high redshift quasars, galaxies and clusters. The results also imply that the value of $G\mu$ could be lowered from its canonical value of $G\mu \simeq 10^{-6}$ without jeopardizing early structure formation.

Acknowledgements

This work is supported in part by the US Department of Energy under Grant DE-FG0291ER40688, Task A (Brown) and by the Canadian NSERC under Grant 580441 (UBC). One of us (R.B.) is grateful to Richhild Moessner for useful conversations, and to Bill Unruh for his warm hospitality at UBC, where this article was written up. This work is based on the senior thesis of A.A. at Brown University.

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